Second-Order Statistics for Diversity-Combining of Non-identical, Unbalanced, Correlated Weibull Signals

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***Abstract*— This paper derives exact expressions for the level crossing rate (LCR) and average fade duration (AFD) of a two-branch selection, equal gain and maximal-ratio combining systems in a Weibull fading environment. The expressions apply to unbalanced, non-identical, correlated diversity channels and have been validated by specializing the general results to some particular cases whose solutions are known. In addition, new closed-form solutions for some special cases are obtained.**

***Index Terms*— Average fade duration, equal-gain combining, maximal-ratio combining, level crossing rate, selection combining, Weibull fading environment.**

I. Introduction

Diversity-combining techniques constitute an effective means to combat the deleterious effects of multipath fading on the performance of wireless communication systems. This performance can be evaluated by several means including the LCR and AFD. LCR and AFD are important second-order statistical quantities, which have been extensively explored in the literature . In [1], LCR and AFD expressions of selection-combining (SC), maximal-ratio combining (MRC), and equal gain combining (EGC) for balanced, dual-branch diversity in correlated Rayleigh channels were presented. The case of unbalanced, non-identical, and correlated Hoyt channels was shown in [2]. Some results involving the independent but non-identical Weibull channels for SC diversity have been presented in [3]. This paper derives exact expressions for the LCR and AFD of a two-branch selection, equal gain and maximal-ratio combining systems in a Weibull fading environment. The expressions apply to unbalanced, non-identical, correlated diversity channels and have been validated by specializing the general results to some particular cases whose solutions are known. In addition, new closed-form solutions for some special cases are obtained.

This paper is organized as follows. Section II establishes the model of Weibull fading channels. Section III derives the Joint Bidimensional Envelope-Phase Weibull density. Some key statistics involving the branch envelopes and their time derivatives are derived in section IV. Relying upon these statistics, the general exact LCR and AFD expressions are also presented. Section V derives the conditional statistics of the received complex signal at the ith branch. Section VI computes the means and the variances for each diversity system. Section VII shows some numerical plots, and finally Section VIII draws some conclusions.

II. Preliminaries

The Weibull distribution is an empirical distribution, which was first proposed aiming at applications in reliability engineering. It has also found use in wireless communications to model the fading envelope. Due to the lack of a theoretical basis [4], the application of the Weibull distribution in wireless communications has been limited to the first order statistics of the fading signal. In [5], a very simple physical model for the Weibull distribution was proposed. In essence, in the proposed model the received signal Z_i at the ith branch ($i = 1, 2$) can be represented in a complex form as

$$Z_i = R_i^{\frac{\alpha_i}{2}} \exp(j\Theta_i) = X_i + jY_i \tag{1}$$

where the in-phase component X_i and the quadrature component Y_i are independent zero-mean Gaussian variates with identical variances σ_i^2, and α_i stands for the fading parameter. Consider that the random phase $\theta_i = \arctan^{-1}(\frac{Y_i}{X_i})$ is uniformly distributed in [0,2π). The probability density function (PDF) $p_{R_i}(\cdot)$ of the envelope R_i is given by

$$p_{R_i}(r_i) = \frac{\alpha_i r_i^{\alpha_i - 1}}{\Omega_i} \exp\left(-\frac{r_i^{\alpha_i}}{\Omega_i}\right) \tag{2}$$

where $\Omega_i = \mathrm{E}\left(R_i^{\alpha_i}\right)$. As the value of fading parameter α_i increases, the severity of the fading decreases, while for the especial case of $\alpha_i = 2$, (2) reduces to well-known Rayleigh PDF. The corresponding cumulative distribution function (CDF) of R_i is given by

$$P_{R_i}(r_i) = 1 - \exp\left(-\frac{r_i^{\alpha_i}}{\Omega_i}\right) \tag{3}$$

From (1), it can be seen that the resulting envelope is given as the modulus of the multipath Rayleigh component R_{li} to the power $2/\alpha_i > 0$. Hence, the relation between R_i and R_{li} can be expressed as

$$R_i = R_{li}^{\frac{2}{\alpha_i}} \tag{4}$$

III. The Joint Bidimensional Envelope-Phase Weibull Density

The joint bidimensional envelope-phase Weibull density (JBEPWD) can be obtained by capitalizing on some results already available in the literature for the Rayleigh distribution. The joint bidimensional envelope-phase Rayleigh distribution $p_{R_{l1},R_{l2},\Theta_1,\Theta_2}(r_{l1}, r_{l2}, \theta_1, \theta_2)$ is given by [6, Eq. 7.51]. From (4), follows that $R_1^{\alpha_1} = R_{l1}^2$ and $R_2^{\alpha_2} = R_{l2}^2$, hence the

JBEPWD can be found as

$$p_{R_1,R_2,\Theta_1,\Theta_2}(r_1,r_2,\theta_1,\theta_2) = |J|^{-1}\frac{r_1^{\frac{\alpha_1}{2}} r_2^{\frac{\alpha_2}{2}}}{4\pi^2|\mathbf{\Lambda}|^{\frac{1}{2}}}$$
$$\times \exp\left[-\frac{1}{2|\mathbf{\Lambda}|^{\frac{1}{2}}}\left(r_1^{\alpha_1}\frac{\Omega_1}{2}+r_2^{\alpha_2}\frac{\Omega_2}{2}\right)\right]$$
$$\times \exp\left[-\frac{1}{2|\mathbf{\Lambda}|^{\frac{1}{2}}}\left(-\mu_1 r_1^{\frac{\alpha_1}{2}} r_2^{\frac{\alpha_2}{2}}\sqrt{\Omega_1\Omega_2}\cos\theta_{12}\right)\right]$$
$$\times \exp\left[-\frac{1}{2|\mathbf{\Lambda}|^{\frac{1}{2}}}\left(\mu_2 r_1^{\frac{\alpha_1}{2}} r_2^{\frac{\alpha_2}{2}}\sqrt{\Omega_1\Omega_2}\sin\theta_{12}\right)\right] \quad (5)$$

where $|\cdot|$ denotes the determinant operator, $|J|$ is the Jacobian of the transformation given by $|J| = \frac{4}{\alpha_1\alpha_2} r_1^{1-\frac{\alpha_1}{2}} r_2^{1-\frac{\alpha_2}{2}}$, θ_{12} is defined as $\theta_{12} = \theta_2 - \theta_1$, and $\mathbf{\Lambda}$ is the covariance matrix given by

$$\mathbf{\Lambda} = \frac{1}{2}\begin{bmatrix} \Omega_1 & 0 & \mu_1\sqrt{\Omega_1\Omega_2} & -\mu_2\sqrt{\Omega_1\Omega_2} \\ 0 & \Omega_1 & \mu_2\sqrt{\Omega_1\Omega_2} & \mu_1\sqrt{\Omega_1\Omega_2} \\ \mu_1\sqrt{\Omega_1\Omega_2} & \mu_2\sqrt{\Omega_1\Omega_2} & \Omega_2 & 0 \\ -\mu_2\sqrt{\Omega_1\Omega_2} & \mu_1\sqrt{\Omega_1\Omega_2} & 0 & \Omega_2 \end{bmatrix} \quad (6)$$

The correlation coefficients μ_1, μ_2 are defined as $\mu_1 = \frac{\mathrm{E}(X_1X_2)-\mathrm{E}(X_1)\mathrm{E}(X_2)}{\sqrt{\mathrm{var}(X_1)\mathrm{var}(X_2)}} = \frac{\mathrm{E}(Y_1Y_2)-\mathrm{E}(Y_1)\mathrm{E}(Y_2)}{\sqrt{\mathrm{var}(Y_1)\mathrm{var}(Y_2)}}$ and $\mu_2 = -\frac{\mathrm{E}(X_1Y_2)-\mathrm{E}(X_1)\mathrm{E}(Y_2)}{\sqrt{\mathrm{var}(X_1)\mathrm{var}(Y_2)}} = \frac{\mathrm{E}(Y_1X_2)-\mathrm{E}(Y_1)\mathrm{E}(X_2)}{\sqrt{\mathrm{var}(Y_1)\mathrm{var}(X_2)}}$. The orthogonality between phase and quadrature components implies $\mathrm{E}(X_1Y_1) = \mathrm{E}(X_2Y_2) = 0$ ($\mathrm{E}(\cdot)$ denotes the expectation operator). After some algebraic manipulations, the JBEPWD can be found as

$$p_{R_1,R_2,\Theta_1,\Theta_2}(r_1,r_2,\theta_1,\theta_2) = \frac{\alpha_1\alpha_2 r_1^{\alpha_1-1}r_2^{\alpha_2-1}}{4\pi^2\Omega_1\Omega_2(1-\rho^2)}$$
$$\times \exp\left(-\frac{r_1^{\alpha_1}\Omega_1 + r_2^{\alpha_2}\Omega_2}{(1-\rho^2)\Omega_1\Omega_2}\right)$$
$$\times \exp\left(-\frac{\left(2r_1^{\frac{\alpha_1}{2}} r_2^{\frac{\alpha_2}{2}}\sqrt{\Omega_1\Omega_2}\right)(\mu_1\cos\theta_{12}+\mu_2\sin\theta_{12})}{(1-\rho^2)\Omega_1\Omega_2}\right) \quad (7)$$

where $\rho^2 = \mu_1^2 + \mu_2^2$.

IV. Average LCR and AFD

The average LCR is defined as the average number of times per unit duration that the envelope of a fading channel crosses a given value in the negative direction and it can be evaluated as

$$N_R(r) = \int_0^\infty \dot{r} p_{R,\dot{R}}(r,\dot{r})d\dot{r} \quad (8)$$

where $p_{R,\dot{R}}(\cdot,\cdot)$ is the joint PDF of R and its time derivative $\dot{R}$. The AFD corresponds to the average length of time the envelope remains under a certain value once it crosses it in the negative direction and can be obtained as

$$T_R(r) = \frac{P_R(r)}{n_R(r)} \quad (9)$$

where $P_R(\cdot)$ is the CDF of the envelope R. In the following, (8) and (9) shall be calculated for the dual-branch, correlated, non-identical, unbalanced Weibull fading environment using the SC, EGC and MRC.

A. Diversity Systems

The output envelope and its output time derivative in the SC, EGC and MRC combining system are given, respectively, by

$$R = \begin{cases} \max\{R_1,R_2\} & \text{SC} \\ \frac{R_1+R_2}{\sqrt{2}} & \text{EGC} \\ \sqrt{R_1^2+R_2^2} & \text{MRC} \end{cases} \quad (10a)$$

$$\dot{R} = \begin{cases} \begin{matrix}\dot{R}_1 & R_1 \geq R_2 \\ \dot{R}_2 & R_1 < R_2\end{matrix} & \text{SC} \\ \frac{\dot{R}_1+\dot{R}_2}{\sqrt{2}} & \text{EGC} \\ \frac{R_1\dot{R}_1+R_2\dot{R}_2}{\sqrt{R_1^2+R_2^2}} & \text{MRC} \end{cases} \quad (10b)$$

In [7] it has been shown that $\dot{R}_i$ given R_i and Θ_i is a zero-mean Gaussian variate. On the other hand, when the branches are correlated the mean of the conditional Gaussian distribution of $\dot{R}$ given R_i's and Θ_i's is not zero, although $\dot{R}_i$ given R_i and Θ_i still is a zero-mean Gaussian variate. From the above, and using the properties of the conditional probability, the following can be written

$$p_{\dot{R},R_1,R_2,\Theta_1,\Theta_2}(\dot{r},r_1,r_2,\theta_1,\theta_2) = p_{\dot{R}|R_1,R_2,\Theta_1,\Theta_2}(\dot{r}|r_1,r_2,\theta_1,\theta_2) \times p_{R_1,R_2,\Theta_1,\Theta_2}(r_1,r_2,\theta_1,\theta_2) \quad (11)$$

The expression in (11) is used since $p_{R_1,R_2,\Theta_1,\Theta_2}(r_1,r_2,\theta_1,\theta_2)$ is given by (7) and the conditional joint distribution $p_{\dot{R}|R_1,R_2,\Theta_1,\Theta_2}(\dot{r}|r_1,r_2,\theta_1,\theta_2)$ is Gaussian with mean $m_{\dot{R}}(r_1,r_2,\theta_1,\theta_2)$ and variance $\sigma^2_{\dot{R}}(r_1,r_2,\theta_1,\theta_2)$, hence

$$\vartheta(r_1,r_2) \triangleq \int_0^\infty \dot{r}\, p_{\dot{R}|R_1,R_2,\Theta_1,\Theta_2}(\dot{r}|r_1,r_2,\theta_1,\theta_2)\, d\dot{r} = \frac{\sigma_{\dot{R}}(r_1,r_2,\theta_1,\theta_2)}{\sqrt{2\pi}}\exp\left(-\frac{m^2_{\dot{R}}(r_1,r_2,\theta_1,\theta_2)}{2\sigma^2_{\dot{R}}(r_1,r_2,\theta_1,\theta_2)}\right) + \frac{m_{\dot{R}}(r_1,r_2,\theta_1,\theta_2)}{2}\left(1+\mathrm{erf}\left(\frac{m_{\dot{R}}(r_1,r_2,\theta_1,\theta_2)}{\sqrt{2}\sigma_{\dot{R}}(r_1,r_2,\theta_1,\theta_2)}\right)\right) \quad (12)$$

where $\mathrm{erf}(\cdot)$ is the error function. To alleviate the notation, we denote $m_{\dot{R}}(r_1,r_2,\theta_1,\theta_2)$ and $\sigma^2_{\dot{R}}(r_1,r_2,\theta_1,\theta_2)$ by $m_{\dot{R}}$ and $\sigma^2_{\dot{R}}$, respectively. The trick part of the problem is the determination of $m_{\dot{R}}$ and $\sigma^2_{\dot{R}}$ for each combining scheme. For the moment, assume that these quantities are known. The general formula to the LCR for the SC, MRC, and EGC is given by [2, Eq. 10]. Using this reference and the equation

(12), the LCR can be obtained as

$$N_R(r) = \begin{cases} \int_0^{2\pi}\int_0^{2\pi}\int_0^{r} \vartheta_1(r_1,r) p_{R_1,R_2,\Theta_1,\Theta_2}(r_1,r,\theta_1,\theta_2) \\ \qquad dr_1 d\theta_1 d\theta_2 \\ +\int_0^{2\pi}\int_0^{2\pi}\int_0^{r} \vartheta_2(r,r_2) p_{R_1,R_2,\Theta_1,\Theta_2}(r,r_2,\theta_1,\theta_2) \\ \qquad dr_2 d\theta_1 d\theta_2 \\ \qquad \text{SC} \\ \int_0^{2\pi}\int_0^{2\pi}\int_0^{\sqrt{2}r} \sqrt{2}\,\vartheta(r_1,\sqrt{2}r-r_1) \\ p_{R_1,R_2,\Theta_1,\Theta_2}\left(r_1,\sqrt{2}r-r_1,\theta_1,\theta_2\right) dr_1 d\theta_1 d\theta_2 \quad \text{EGC} \\ \int_0^{2\pi}\int_0^{2\pi}\int_0^{r} \frac{r}{\sqrt{r^2-r_1^2}}\,\vartheta(r_1,\sqrt{r^2-r_1^2}) \\ p_{R_1,R_2,\Theta_1,\Theta_2}\left(r_1,\sqrt{r^2-r_1^2},\theta_1,\theta_2\right) dr_1 d\theta_1 d\theta_2 \ \text{MRC} \end{cases} \tag{13}$$

where $\vartheta_i(r_1,r_2) = \frac{\sigma_{\dot{R}_i}}{\sqrt{2\pi}} e^{-\frac{m^2_{\dot{R}_i}}{2\sigma^2_{\dot{R}_i}}} + \frac{m_{\dot{R}_i}}{2}\left(1+\text{erf}\left(\frac{m_{\dot{R}_i}}{\sqrt{2}\sigma_{\dot{R}_i}}\right)\right)$, $(i = 1,2)$, and the CDF can be obtained as [8]

$$P_R(r) = \int_0^{\gamma_1}\int_0^{\gamma_2}\int_0^{2\pi}\int_0^{2\pi} p_{R_1,R_2,\Theta_1,\Theta_2}(r_1,r_2,\theta_1,\theta_2) \, d\theta_1 d\theta_2 dr_2 dr_1 \tag{14}$$

where

$$\begin{cases} \gamma_1 = \gamma_2 = r \quad \text{for SC} \\ \gamma_1 = \sqrt{2}r, \gamma_2 = \sqrt{2}r - r_1 \quad \text{for EGC} \\ \gamma_1 = r\,,\ \gamma_2 = \sqrt{r^2-r_1^2} \quad \text{for MRC} \end{cases} \tag{15}$$

The AFD follows directly from (9), (13) and (14).

From (10), the conditional means and variances for each combining scheme can be obtained as

1) Selection Combining:

- If $R_1 \geq R_2$

$$m_{\dot{R}} = m_{\dot{R}_1} \tag{16}$$
$$\sigma^2_{\dot{R}} = \sigma^2_{\dot{R}_1} \tag{17}$$

- If $R_1 < R_2$

$$m_{\dot{R}} = m_{\dot{R}_2} \tag{18}$$
$$\sigma^2_{\dot{R}} = \sigma^2_{\dot{R}_2} \tag{19}$$

2) Equal-Gain Combining:

$$m_{\dot{R}} = \frac{m_{\dot{R}_1} + m_{\dot{R}_1}}{\sqrt{2}} \tag{20}$$

$$\sigma^2_{\dot{R}} = \frac{\sigma^2_{\dot{R}_1} + \sigma^2_{\dot{R}_2} + 2\sigma_{\dot{R}_1,\dot{R}_2}}{2} \tag{21}$$

where $\sigma_{\dot{R}_1,\dot{R}_2}$ stands for the covariance between $\dot{R}_1$ and $\dot{R}_2$ given $\mathbf{Z}$.

3) Maximal-Ratio Combining:

$$m_{\dot{R}} = \frac{r_1\, m_{\dot{R}_1} + r_2\, m_{\dot{R}_2}}{\sqrt{r_1^2+r_2^2}} \tag{22}$$

$$\sigma^2_{\dot{R}} = \frac{r_1^2\sigma^2_{\dot{R}_1} + r_2^2\sigma^2_{\dot{R}_2} + 2r_1r_2\sigma_{\dot{R}_1,\dot{R}_2}}{r_1^2+r_2^2} \tag{23}$$

V. Conditional Statistics of $\dot{Z}_i$

The aim of this section and section VI is to find the mean $m_{\dot{R}_i}$, the variance $\sigma^2_{\dot{R}_i}$, and the covariance $\sigma_{\dot{R}_i,\dot{R}_j}$ of the conditional Gaussian density $p_{\dot{R}|R_1,R_2,\Theta_1,\Theta_2}(\dot{r}|r_1,r_2,\theta_1,\theta_2)$ for each combining technique. Before obtaining these statistics, let us derive the conditional statistics of $\dot{Z}_i$ which is a crucial step for solving the problem addressed here.

Let $\dot{\mathbf{Z}} = [\dot{Z}_1\dot{Z}_2]$ and $\mathbf{Z} = [Z_1Z_2]$ column matrices of $\dot{Z}_i = \dot{Z}_i(t)$ and $Z_i = Z_i(t)$, mutually correlated zero-mean Gaussian processes. The complex covariance matrix, $\Phi(4\times4)$, between them is defined as [1]

$$\Phi = \frac{1}{2}\,\text{E}\left[\left(\begin{array}{c}\dot{\mathbf{Z}}\\ \mathbf{Z}\end{array}\right)^*\left(\begin{array}{c}\dot{\mathbf{Z}}\\ \mathbf{Z}\end{array}\right)^T\right] \triangleq \left[\begin{array}{cc}\mathbf{a} & \mathbf{c}\\ \mathbf{c}^H & \mathbf{b}\end{array}\right] \tag{24}$$

where $(\cdot)^*$ denotes the conjugate operator, $(\cdot)^T$ the transpose matrix, and $(\cdot)^H$ the hermitian matrix. Defining $\rho_{ij}(\tau)$ as the complex crosscorrelation function between ith and jth branches, $\dot{\rho_{ij}} = \left.\frac{d\rho_{ij}(\tau)}{d\tau}\right|_{\tau=0}$, $\ddot{\rho_{ij}} = \left.\frac{d^2\rho_{ij}(\tau)}{d\tau^2}\right|_{\tau=0}$, $\rho_{ij} \triangleq \rho_{ij}(0)$, the matrices $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ can be expressed as

$$\mathbf{a} = \frac{1}{2}\left[\begin{array}{cc} -\ddot{\rho_{11}}\Omega_1 & -\ddot{\rho_{12}}\sqrt{\Omega_1\Omega_2} \\ -\ddot{\rho}^*_{12}\sqrt{\Omega_1\Omega_2} & -\ddot{\rho_{11}}\Omega_2 \end{array}\right] \tag{25}$$

$$\mathbf{b} = \frac{1}{2}\left[\begin{array}{cc} \Omega_1 & \rho_{12}\sqrt{\Omega_1\Omega_2} \\ \rho^*_{12}\sqrt{\Omega_1\Omega_2} & \Omega_2 \end{array}\right] \tag{26}$$

$$\mathbf{c} = \frac{1}{2}\left[\begin{array}{cc} 0 & -\dot{\rho_{12}}\sqrt{\Omega_1\Omega_2} \\ \dot{\rho}^*_{12}\sqrt{\Omega_1\Omega_2} & 0 \end{array}\right] \tag{27}$$

Note that the diagonal elements in the matrix $\mathbf{c}$ are null, because for a stationary process the correlation between the process and its time derivative is always null at $\tau = 0$ ($\dot{\rho_{12}} = 0$) [9].

Applying the matrix theory described on [10, pp. 495-496], the conditional density of $\dot{\mathbf{Z}}$ given $\mathbf{Z}$ is Gaussian distributed with mean matrix $\mathbf{M}$ and covariance matrix $\mathbf{\Delta}$ given by

$$\mathbf{M} = \left[\begin{array}{c} \text{E}(\dot{Z}_1|\mathbf{Z}) \\ \text{E}(\dot{Z}_2|\mathbf{Z}) \end{array}\right] = (\mathbf{c}\mathbf{b}^{-1})^*\,\mathbf{Z} \tag{28}$$

$$\mathbf{\Delta} = \left[\begin{array}{cc} \text{Var}(\dot{Z}_1|\mathbf{Z}) & \text{Cov}(\dot{Z}_1,\dot{Z}_2|\mathbf{Z}) \\ \text{Cov}(\dot{Z}_1,\dot{Z}_2|\mathbf{Z})^* & \text{Var}(\dot{Z}_2|\mathbf{Z}) \end{array}\right] = \mathbf{a} - \mathbf{c}\mathbf{b}^{-1}\mathbf{c}^H \tag{29}$$

where $\text{Var}(\cdot)$ and $\text{Cov}(\cdot)$ denote variance and covariance, respectively.

Substituting (25), (26), (27) into (28) and (29), the conditional statistics of $\dot{Z}_i$ given $\mathbf{Z}$ is derived.

VI. Means and Variances of $\dot{R}_i s$

This section relates the complex variates $\dot{Z}_i$'s with the real variates $\dot{R}_i$'s. In the previous section, the mean, the variance, and the covariance of $\dot{Z}_i$'s given $\mathbf{Z}$ have been determined. Now the mean, the variance, and the covariance of the R_i's will be calculated given $\mathbf{Z}$. Using $\dot{R}_i = \frac{2}{\alpha_i} R_i^{1-\frac{\alpha_i}{2}} \operatorname{Re}[\dot{Z}_i \exp(-j\theta_i)]$, where $\operatorname{Re}[\cdot]$ denotes the real part of a complex number, we can write

$$m_{\dot{R}_i} \triangleq \mathrm{E}(\dot{R}_i|\mathbf{Z}) = \frac{2}{\alpha_i} R_i^{1-\frac{\alpha_i}{2}} \operatorname{Re}[\mathrm{E}(\dot{Z}_i|\mathbf{Z}) \exp(-j\theta_i)] \quad (30)$$

$$\sigma^2_{\dot{R}_i} \triangleq \operatorname{Var}(\dot{R}_i|\mathbf{Z}) = \frac{2R_i^{2-\alpha_i}}{\alpha_i^2} \operatorname{Var}(\dot{Z}_i|\mathbf{Z}) \quad (31)$$

$$\sigma_{\dot{R}_i,\dot{R}_j} \triangleq \operatorname{Cov}(\dot{R}_i,\dot{R}_j|\mathbf{Z}) = \frac{2}{\alpha_i\alpha_j} R_i^{1-\frac{\alpha_i}{2}} R_j^{1-\frac{\alpha_j}{2}} \times \operatorname{Re}[e^{j\theta_{12}} \operatorname{Cov}(\dot{Z}_i,\dot{Z}_j|\mathbf{Z})^*] \quad (32)$$

Using the results from (28) and (29), the following mean, variance and covariance are obtained

$$m_{\dot{R}_1} = \frac{2}{\alpha_1} \frac{1}{1-|\rho_{12}|^2} \left[r_1 \operatorname{Re}[\rho_{12}\dot{\rho}_{12}^*] - r_1^{1-\frac{\alpha_1}{2}} r_2^{\frac{\alpha_2}{2}} \sqrt{\frac{\Omega_1}{\Omega_2}} \operatorname{Re}[\dot{\rho}_{12}^* e^{j\theta_{12}}] \right] \quad (33)$$

$$m_{\dot{R}_2} = \frac{2}{\alpha_2} \frac{1}{1-|\rho_{12}|^2} \left[r_1^{\frac{\alpha_1}{2}} r_2^{1-\frac{\alpha_2}{2}} \sqrt{\frac{\Omega_2}{\Omega_1}} \operatorname{Re}[\dot{\rho}_{12}^* e^{j\theta_{12}}] - r_2 \operatorname{Re}[\rho_{12}\dot{\rho}_{12}^*] \right] \quad (34)$$

$$\sigma^2_{\dot{R}_i} = -\frac{2\,\Omega_i\, r_i^{2-\alpha_i}}{\alpha_i^2} \left[\ddot{\rho_{11}} + \frac{|\dot{\rho_{12}}|^2}{1-|\rho_{12}|^2} \right], \ i = 1, 2 \quad (35)$$

$$\sigma_{\dot{R}_1,\dot{R}_2} = -\frac{2\sqrt{\Omega_1\Omega_2}}{\alpha_1\alpha_2} r_1^{1-\frac{\alpha_1}{2}} r_2^{1-\frac{\alpha_2}{2}} \times \operatorname{Re}\left[\left(\ddot{\rho_{12}} + \frac{(\dot{\rho_{12}})^2 \rho_{12}^*}{1-|\rho_{12}|^2} \right)^* e^{j\theta 12} \right] \quad (36)$$

A. Special Cases

Observe that for the Rayleigh case, $\alpha_i = 2$ $(i = 1, 2)$, and balanced channels $\Omega_1 = \Omega_2 = 2\sigma^2$, the equations from (16) to (23) reduce in a *exact manner* to those of [1, Eqs.26 and 27]. In particular for $\alpha_i = 2$ and unbalanced case, these expressions presented here *generalize* the results presented in [1] for the Rayleigh channels.

VII. Numerical Results

The expressions obtained for the LCR and AFD are general and can be applied to any type of diversity (space, frequency or time). In this section, we assume space diversity at the mobile station in order to apply the expressions above derived. For incoming multipath waves having equal amplitude and independent phases, the crosscorrelation functions are given by [1]

$$\rho_{11}(\tau) = J_0(2\pi f_m \tau) \quad (37)$$

$$\rho_{12}(\tau) = J_0\left(2\pi\sqrt{(f_m\tau)^2 + (d/\lambda)^2 - 2(f_m\tau)} \times \sqrt{d/\lambda\ \cos(\beta)}\right) \quad (38)$$

where $J_0(\cdot)$ is the zero-order Bessel function, λ is the carrier wavelength, f_m is the maximum Doppler shift in Hz, d is the antenna spacing, and $\beta \in [0, 2\pi]$ is the angle between the antenna axis and the direction of the vehicle motion in radians. The corresponding correlation coefficients can be calculated as

$$\rho_{11} = 1 \quad (39)$$

$$\rho_{12} = J_0(2\pi d/\lambda) \quad (40)$$

$$\dot{\rho}_{12} = 2\pi f_m \cos(\beta)\, J_1(2\pi d/\lambda) \quad (41)$$

$$\ddot{\rho}_{12} = (2\pi f_m)^2 \left\{ \frac{J_1(2\pi d/\lambda)}{2\pi d/\lambda} \cos(2\beta) - \cos^2(\beta)\, J_0(2\pi d/\lambda) \right\} \quad (42)$$

$$\ddot{\rho}_{11} = -2(\pi f_m)^2 \quad (43)$$

where $J_1(\cdot)$ is the first-order Bessel function. The parameters μ_1 and μ_2 used in JBEPWD were expressed as $\mu_1 = \rho_{12}$ and $\mu_2 = 0$ since the angular frequency separation is null [11].

Fig. 1 shows the normalized LCR (left axis), N_R/f_m, and AFD (right axis), $T_R f_m$, as a function of the envelope, for the SC, EGC and MRC. The following arbitrary parameters have been used: $d/\lambda = 0.06$, $\alpha_i = 3$, $\beta = 0$ and $\beta = \pi/2$. The branches are considered balanced and identical. The curves without diversity reception have not been included for the sake of simplicity, but the use of diversity reception reduces the LCR, particulary for deep fades. Figs. 2 and 3 show the normalized LCR and AFD, respectively, as a function of the parameter d/λ, for the SC, EGC and MRC. It has been considered an envelope level at $r = -20$ dB, identical fading parameters $\alpha_i = 3$, two different antenna angles $\beta = 0$ and $\beta = \pi/2$, and balanced channels. It can be seen in Fig. 2 that as the antenna spacing becomes large, the LCR decreases, becoming oscillatory and convergent. Fig. 2 also shows that the MRC has the smaller LCR in both cases of antenna angles. It can be seen in Fig. 3 that the shape of the AFD curves for the SC, EGC and MRC are loosely dependent on the antenna spacing when $\beta = \pi/2$.

VIII. Conclusions

Exact formulas for level crossing rate and average fade duration of the dual branch SC, EGC and MRC techniques in a unbalanced, non-identical, and correlated Weibull fading environment were presented. Furthermore, this paper provides the joint Weibull bidimensional envelope-phase distribution. These formulas have been validated by specializing the general results to some particular case whose solutions are known.

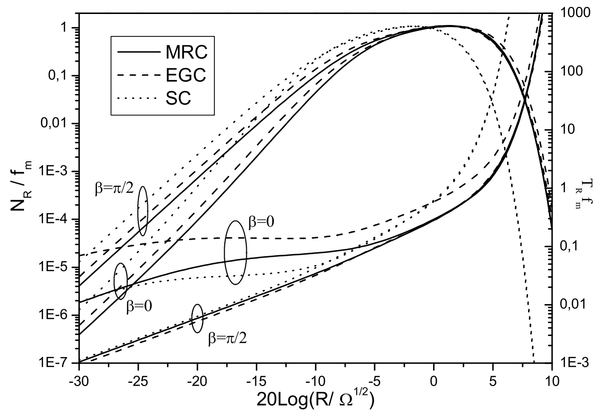

Fig. 1. LCR and AFD for $d/\lambda = 0.06$ for SC, EGC and MRC considering $\alpha_i = 3$ and identical Weibull-fading channels.

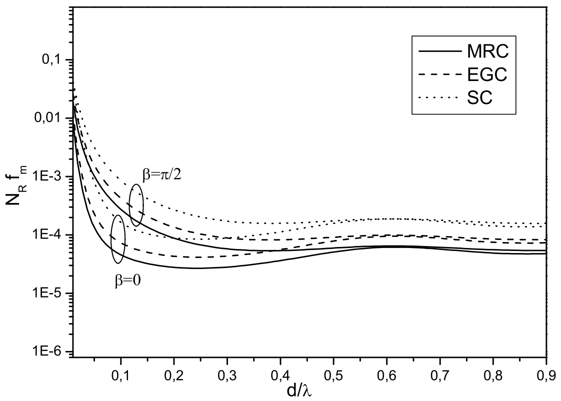

Fig. 2. LCR for r = -20dB for SC, EGC and MRC.

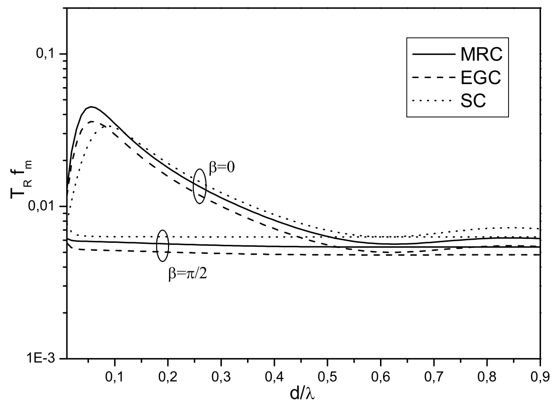

Fig. 3. AFD for r = -20dB for SC, EGC and MRC.

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